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DETERMINATION OF THE FLOW FIELD IN A REGION ADJACENT TO A STREAMLINED BODY AT SMALL REYNOLDS NUMBER

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A steady flow of a viscous incompressible fluid past a body of finite dimensions and a sufficiently smooth boundary is studied at small Reynolds number 2λ. The asymptotic behavior of the velocity of flow is investigated at the distance $R \leqslant \lambda^{-1}$ from the body. It is proved that the velocity can be represented in the form of a sum of the velocity in the linear Oseen approximation, which is an analytic function of the Reynolds number, and of a section of the asymptotic series in terms of the Reynolds number. The coefficients of this section are determined as solutions of certain boundary value problems in the Stokes approximation. An explicit formula is obtained for the flow past a sphere, which coincides with the known internal expansion due to Proudman and Pearson.

1. A steady flow of a viscous incompressible fluid past a body B is described by a system of Navier–Stokes equations with the boundary conditions

$$\mathbf{u}\cdot\nabla\mathbf{u} + \operatorname{grad} p = \frac{1}{2\lambda}\Delta\mathbf{u}, \quad \operatorname{div}\mathbf{u} = 0 \tag{1.1}$$

$$\mathbf{u}|_S = \mathbf{u}_0, \quad \lim_{|\mathbf{x}|\to\infty} \mathbf{u} = \mathbf{u}_\infty = (1, 0, 0)$$

(S denotes the surface of the body). We obtain the following boundary value problem for the perturbation velocity $\mathbf{v} = \mathbf{u} - \mathbf{u}_\infty$:

$$\Delta\mathbf{v} - 2\lambda\frac{\partial\mathbf{v}}{\partial x_1} - 2\lambda \operatorname{grad} p = 2\lambda\mathbf{v}\cdot\nabla\mathbf{v}, \qquad \operatorname{div}\mathbf{v} = 0 \tag{1.2}$$

$$\mathbf{v}|_S = \mathbf{v}_0, \quad \lim_{|\mathbf{x}|\to\infty} \mathbf{v} = 0$$

We know [1] that if the surface S and function $\mathbf{v}_0 = \mathbf{u}_0 - \mathbf{u}_\infty$ both satisfy certain conditions of smoothness, a solution of the boundary value problem in question exists, for any value of the Reynolds number in the class of functions with the bounded Dirichlet integral

$$\int_D |\nabla\mathbf{u}|^2 d\mathbf{x} < C, \quad D = R^3 \setminus B$$

In what follows, we shall use the following two Babenko's theorems [2].

Theorem 1. If $S \in C^{2+\delta}$, $\delta > 0$, $\mathbf{v}_0 \in C^2\,[S]$, then a Reynolds number Re_* exists such that the solution of the boundary value problem (1.1) is given by the formulas

$$\mathbf{v}(\mathbf{x}, \lambda) = \sum_{k=0}^{\infty} (2\lambda)^k \mathbf{v}^{(k)}(\mathbf{x}, \lambda), \quad p(\mathbf{x}, \lambda) = \sum_{k=0}^{\infty} (2\lambda)^k p^{(k)}(\mathbf{x}, \lambda) \tag{1.3}$$

which converge when $2\lambda < \mathrm{Re}_*$.

If S and $\mathbf{v}_0$ appear in the compact families, then $\inf \mathrm{Re}_* > 0$ where the lower edge is taken along these families.

Theorem 2. Let the conditions of Theorem 1 hold. Then a Reynolds number Re_u $(0 < \mathrm{Re}_u < \infty)$ exists such that for $2\lambda < \mathrm{Re}_u$ the solution of the problem (1.1) with a bounded Dirichlet integral is unique.

The functions $\mathbf{v}^{(0)}(\mathbf{x}, \lambda)$, $p^{(0)}(\mathbf{x}, \lambda)$ in (1.3) represent a solution of the boundary value problem for a homogeneous system of Oseen equations (which is obtained from the system (1.2) by discarding the nonlinear term $2\lambda \mathbf{v}\cdot\nabla\mathbf{v}$) with inhomogeneous boundary conditions from (1.2), and $\mathbf{v}^{(k)}(\mathbf{x}, \lambda)$, $p^{(k)}(\mathbf{x}, \lambda)$ $(k \geqslant 1)$ is the solution of the inhomogeneous Oseen system

$$\Delta \mathbf{v}^{(k)} - 2\lambda \frac{\partial \mathbf{v}^{(k)}}{\partial x_1} - 2\lambda \operatorname{grad} p^{(k)} = \sum_{i=1}^{3} \sum_{j=0}^{k-1} v_i^{(j)} \frac{\partial \mathbf{v}^{(k-1-j)}}{\partial x_i} \qquad (1.4)$$

$$\operatorname{div} \mathbf{v}^{(k)} = 0$$

with homogeneous boundary conditions.

An expansion of the form (1.3) was used earlier in [3] in the course of proving the existence of solution of a streamlined flow at small Reynolds numbers in the class of functions satisfying the condition $\mathbf{v} = O(|\mathbf{x}|^{-1})$.

2. To obtain an asymptotic expression for the velocity at small Reynolds numbers, of the form

$$\mathbf{v}(\mathbf{x}, \lambda) = \sum_{n=0}^{N} \alpha_n(\lambda)\, \mathbf{U}^{(n)}(\mathbf{x}) + O[\alpha_{N+1}(\lambda)] \qquad (2.1)$$

where $\{\alpha_n(\lambda)\}$ denotes an asymptotic sequence with $\lambda \to 0$, we must expand in λ the coefficients of $\mathbf{v}^{(k)}(\mathbf{x}, \lambda)$ in the first formula of (1.3). We cannot however obtain expansions of the form (2.1) for the whole region D; we can only construct separate expansions in the regions $\{\mathbf{x}: |\mathbf{x}| \ll \lambda^{-1}\} \cap D$ and $\{\mathbf{x}: |\mathbf{x}| \gg \lambda^{-1}\}$ and in the intermediate region. An asymptotic formula for the velocity away from the body valid for any value of the Reynolds number was obtained in [4,5].

According to the first formula of (1.3) we have

$$\mathbf{v}(\mathbf{x}, \lambda) = \mathbf{v}^{(0)}(\mathbf{x}, \lambda) + 2\lambda \mathbf{v}^{(1)}(\mathbf{x}, \lambda) + O(\lambda^2)$$

Let us expand in λ the function

$$\mathbf{v}^{(1)}(\mathbf{x}, \lambda) = \int_D \mathbf{G}(\mathbf{x}, \mathbf{y}; \lambda)\, v_k^{(0)}(\mathbf{y}, \lambda) \frac{\partial \mathbf{v}^{(0)}}{\partial y_k}(\mathbf{y}, \lambda)\, d\mathbf{y} \qquad (2.2)$$

where $\mathbf{G}(\mathbf{x}, \mathbf{y}; \lambda)$ is a Green's matrix. Here and henceforth we assume that repeated indices indicate the summation from 1 to 3. Assuming that

$$\mathbf{G}(\mathbf{x}, \mathbf{y}; \lambda) = \mathbf{H}(\mathbf{x}, \mathbf{y}; \lambda) + \mathbf{V}(\mathbf{x}, \mathbf{y}; \lambda)$$

where $\mathbf{H}(\mathbf{x}, \mathbf{y}; \lambda)$ is the matrix of the fundamental solutions of the Oseen equations, we shall seek the regular part of the Green's matrix $\mathbf{V}(\mathbf{x}, \mathbf{y}; \lambda)$ in the form of the potential of a double layer. Taking into account the fact that by replacing $\mathbf{u}_\infty$ by $-\mathbf{u}_\infty$ we transform the system of Oseen equations into its conjugate, we can write

($\mathbf{n}$ is the unit vector of the exterior normal to the surface S)

$$V_{ij}(\mathbf{x}, \mathbf{y}; \lambda, \mathbf{u}_\infty) = V_{ij}(\mathbf{y}, \mathbf{x}; \lambda, -\mathbf{u}_\infty) = \int_S K_{il}(\mathbf{y}, \mathbf{z}; \lambda, -\mathbf{u}_\infty)\, \varphi_{lj}(\mathbf{z}, \mathbf{x})\, d\sigma_z \quad (i, j = 1, 2, 3) \tag{2.3}$$

$$K_{il} = \left(\frac{\partial H_{il}}{\partial z_k} + \frac{\partial H_{ik}}{\partial z_l} + 2\lambda \delta_{kl} q_i\right) n_i + \lambda n_1 H_{il} - H_{il}$$

$$H_{il} = \delta_{il} \Delta \Phi - \frac{\partial^2 \Phi}{\partial y_i \partial y_l}, \quad \Phi = -\frac{1}{8\pi\lambda} \int_0^{\lambda s} \frac{1 - e^{-t}}{t}\, dt$$

$$s = |\mathbf{y} - \mathbf{z}| - y_1 + z_1, \quad q_i = \frac{1}{2\lambda} \frac{\partial}{\partial y_i} \left(\frac{1}{4\pi |\mathbf{y} - \mathbf{z}|}\right)$$

where the potential was constructed using the method given by Babenko (*). When $\lambda = 0$, this construction yields an unique solution of the integral equation for the potential density

$$\frac{1}{2} \varphi_{lj}(\mathbf{x}, \mathbf{z}) + \int_S K_{lm}(\mathbf{z}, \zeta)\, \varphi_{mj}(\mathbf{x}, \zeta)\, d\sigma_\zeta = -H_{lj}(\mathbf{x}, \mathbf{z}) \tag{2.4}$$

3. The integral (2.2) can be represented in the form of a sum of the following two integrals:

$$\mathbf{I}_1(\mathbf{x}, \lambda) = \int_D \mathbf{H}(\mathbf{x}, \mathbf{y}; \lambda)\, v_k^{(0)}(\mathbf{y}, \lambda) \frac{\partial \mathbf{v}^{(0)}}{\partial y_k}(\mathbf{y}, \lambda)\, d\mathbf{y}$$

$$\mathbf{I}_2(\mathbf{x}, \lambda) = \int_D \mathbf{V}(\mathbf{x}, \mathbf{y}; \lambda)\, v_k^{(0)}(\mathbf{y}, \lambda) \frac{\partial \mathbf{v}^{(0)}}{\partial y_k}(\mathbf{y}, \lambda)\, d\mathbf{y}$$

We know that $\mathbf{v}^{(0)}$ and $\partial \mathbf{v}^{(0)}/\partial y_k$ are both analytic functions of λ when $\lambda = 0$. This follows from the fact that if $\mathbf{v}^{(0)}$ is written in the form of the potential of a double layer,

$$\mathbf{v}^{(0)}(\mathbf{y}) = \int_S \mathbf{K}(\mathbf{y}, \mathbf{z}; \lambda)\, \boldsymbol{\psi}(\mathbf{z})\, d\sigma_z \tag{3.1}$$

then we obtain a linear integral equation for the potential density $\boldsymbol{\psi}(\mathbf{z})$ in which the kernel and the right-hand side are analytic functions of λ when $\lambda = 0$ and the corresponding homogeneous equation has only a trivial solution. Similarly we prove the analyticity of the function $\mathbf{V}(\mathbf{x}, \mathbf{y}; \lambda)$ at $\lambda = 0$ which will be used to determine the asymptotics of the integral $\mathbf{I}_2(\mathbf{x}, \lambda)$.

Let us describe the domain of integration as a unification of the following regions: $D_1 = \{\mathbf{y}: |\mathbf{y}| \leqslant 2a\} \cap D$ ($2a$ is the diameter of the body B), $D_2 = \{\mathbf{y}: 2a \leqslant |\mathbf{y}| \leqslant \lambda^{-1}\}$ and $D_3 = \{\mathbf{y}: |\mathbf{y}| \geqslant \lambda^{-1}\}$. Using the analyticity of $\mathbf{v}^{(0)}$ and $\partial \mathbf{v}^{(0)}/\partial y_k$, we can show that

$$\mathbf{J}_1(\mathbf{x}, \lambda) = \int_{D_1} \mathbf{H}(\mathbf{v}^{(0)} \cdot \nabla \mathbf{v}^{(0)})\, d\mathbf{y} = \mathbf{J}_1(\mathbf{x}, 0) + O(\lambda)$$

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*) Babenko K.I. Theory of perturbation of the stationary flows of a viscous incompressible fluid at small Reynolds numbers. Preprint of the Inst. of Applied Mathematics, AS SSSR, No. 79, 1975.

Consider the integral

$$\mathbf{J}_4 = \int_{D_4} \mathbf{H}(\mathbf{v}^{(0)} \cdot \nabla \mathbf{v}^{(0)})\, d\mathbf{y}, \quad D_4 = D_2 \cup D_3 \tag{3.2}$$

We obtain its asymptotics by expanding the kernel $\mathbf{K}(\mathbf{y}, \mathbf{z}; \lambda)$ in (3.1) into a series in $\mathbf{z}$ and integrating with respect to $\mathbf{z}$, this yields

$$v_i^{(0)}(\mathbf{y}) = A_j H_{ij}(\mathbf{y}) + A \frac{y_i}{|\mathbf{y}|^3} + A_{jk} \frac{\partial H_{ij}}{\partial y_k}(\mathbf{y}) + \ldots, \quad A_j = 2\lambda F_j^{(0)} \tag{3.3}$$

where $F_j^{(0)}$ is the j- th coordinate of the force vector acting on the body B in the Oseen approximation. Substituting the expansion (3.3) into (3.2), we obtain

$$\mathbf{J}_4(\mathbf{x}, \lambda) = \mathbf{j}_4(\mathbf{x}, \lambda) + \mathbf{l}_4(\mathbf{x}, \lambda)$$

$$j_{4i}(\mathbf{x}, \lambda) = A_l A_m \int_{D_4} H_{ij}(\mathbf{x} - \mathbf{y})\, H_{kl}(\mathbf{y}) \frac{\partial H_{jm}}{\partial y_k}(\mathbf{y})\, d\mathbf{y}$$

$$l_{4i}(\mathbf{x}, \lambda) = l_{4i}(\mathbf{x}, 0) + O(\lambda)$$

Let us represent the integral $\mathbf{j}_4$ in the form of a sum of two integrals $\mathbf{j}_2$ and $\mathbf{j}_3$ taken over the regions D_2 and D_3 respectively. Since

$$|\mathbf{y} - \mathbf{x}| \geqslant |\mathbf{y}| - |\mathbf{x}| \geqslant |\mathbf{y}| - h\lambda^{-1} \geqslant (1 - h)|\mathbf{y}| \quad (0 \leqslant h \leqslant 1)$$

we use the estimates of the fundamental solution uniform in λ

$$|H_{ij}(\mathbf{x})| \leqslant C|\mathbf{x}|^{-1}, \quad |\nabla H_{ij}(\mathbf{x})| \leqslant C|\mathbf{x}|^{-3/2}[s(\mathbf{x}) + 1]^{-1/2}$$

to conclude that

$$|\mathbf{j}_3| \leqslant C_1 \int_{D_3} |\mathbf{y}|^{-7/2}[s(\mathbf{y}) + 1]^{-1/2} d\mathbf{y} \leqslant$$

$$C_2 \int_{\lambda^{-1}}^{\infty} \rho^{-3/2} d\rho \int_0^{\pi} [\rho(1 - \cos\theta) + 1]^{-1/2} \sin\theta\, d\theta \leqslant C\lambda$$

where spherical coordinates are used in the last inequality.

Next we consider the integral $\mathbf{j}_2$. Using the expansions

$$H_{kl}(\mathbf{x}, \lambda) = H_{kl}(\mathbf{x}, 0) + \lambda \frac{\partial H_{kl}}{\partial \lambda}(\mathbf{x}, 0) + \lambda^2 \Psi_{kl}(\mathbf{x}, \lambda)$$

$$\frac{\partial H_{jl}}{\partial x_k}(\mathbf{x}, \lambda) = \frac{\partial H_{ij}}{\partial x_k}(\mathbf{x}, 0) + \lambda \frac{\partial^2 H_{ij}}{\partial \lambda\, \partial x_k}(x, 0) + \lambda^2 \Psi_{ijk}(\mathbf{x}, \lambda)$$

$$\Psi_{kl}(\mathbf{x}, \lambda) = O(|\mathbf{x}|), \quad \Psi_{ijk}(\mathbf{x}, \lambda) = O(1)$$

we obtain

$$j_{2i} = A_l A_m \int_{D_2} H_{ij}(\mathbf{x} - \mathbf{y}, 0)\, H_{kl}(\mathbf{y}, 0) \frac{\partial H_{jm}}{\partial y_k}(\mathbf{y}, 0)\, d\mathbf{y} + \tag{3.4}$$

$$B_i \lambda \ln \frac{1}{\lambda} + O(\lambda), \quad B_i = \frac{A_l A_m}{30(8\pi)^2}(7\delta_{1i}\delta_{lm} + 17\delta_{1m}\delta_{il} - 48\delta_{1l}\delta_{im})$$

We note that replacing in the first term of (3.4) the region of integration D_2 by D_4 introduces an error $O(\lambda)$, and the resulting integral will be independent of λ.

Consequently

$$j_{2i} = A_l A_m \int_{D_4} H_{ij}(\mathbf{x}-\mathbf{y},0) H_{kl}(\mathbf{y},0) \frac{\partial H_{jm}}{\partial y_k}(\mathbf{y},0)\, d\mathbf{y} + B_i\lambda \ln\frac{1}{\lambda} + O(\lambda)$$

and we have

$$I_{1i}(x,\lambda) = I_{1i}(x,0) + B_i\lambda\ln\frac{1}{\lambda} + O(\lambda) \qquad (3.5)$$

Let us write $I_{1i}(\mathbf{x},0)$ in terms of the velocity of flow $\mathbf{v}^{(s)}$, in the Stokes approximation, i.e. in terms of the solution of the system

$$\Delta\mathbf{V} - 2\lambda \operatorname{grad} p = 0, \qquad \operatorname{div}\mathbf{v} = 0$$

with the boundary conditions from (1.2). Using the formula given in the paper quoted at the footnote of page 1092

$$\mathbf{v}^{(0)} = \mathbf{v}^{(S)} + O(\lambda), \quad \frac{\partial\mathbf{v}^{(0)}}{\partial y_k} = \frac{\partial\mathbf{v}^{(S)}}{\partial y_k} + O\{\lambda\,|\,\mathbf{y}\,|^{-1/2}[s(\mathbf{y})+1]^{-1/2}\} \qquad (3.6)$$

we can show that

$$I_{1i}(x,\lambda) = \int_D H_{ij}(x-y,0)\, v_k^{(S)} \frac{\partial v_j^{(S)}}{\partial y_k}\, dy + O\left(\lambda\ln\frac{1}{\lambda}\right)$$

and consequently

$$I_{1i}(\mathbf{x},0) = \int_D H_{ij}(\mathbf{x}-\mathbf{y},\ 0)\, v_k^{(S)} \frac{\partial v_j^{(S)}}{\partial y_k}\, d\mathbf{y} \qquad (3.7)$$

4. Consider the integral

$$\mathbf{I}_2(\mathbf{x},\lambda) = \int_D \mathbf{V}(\mathbf{y},\mathbf{x};\lambda,-\mathbf{u}_\infty)\, v_k^{(0)} \frac{\partial\mathbf{v}^{(0)}}{\partial y_k}\, d\mathbf{y}$$

To find the asymptotics of this integral we use the example given in [2] in the course of determining the force acting on a body. From the formulas (7) and (8) it follows that $\mathbf{V}(\mathbf{y},\mathbf{x};\lambda,-\mathbf{u}_\infty)$ is the linear operator $\mathbf{L}$ of the function $-\mathbf{H}(\mathbf{x},\mathbf{z};\mathbf{u}_\infty)$ itself regarded as a function of $\mathbf{z}$, i.e.

$$\mathbf{V}(\mathbf{y},\mathbf{z};\lambda,-\mathbf{u}_\infty) = \mathbf{L}[\mathbf{y};-\mathbf{H}(\mathbf{x},\mathbf{z};\mathbf{u}_\infty,\lambda)] = \mathbf{L}[\mathbf{x};-\mathbf{H}(\mathbf{y},\mathbf{z};-\mathbf{u}_\infty,\lambda)] = \mathbf{L}[\mathbf{x};-\mathbf{H}(\mathbf{z},\mathbf{y};\mathbf{u}_\infty,\lambda)]$$

From this it follows that

$$\mathbf{I}_2(\mathbf{x},\lambda) = \int_D \mathbf{L}[\mathbf{x};-\mathbf{H}(\mathbf{z},\mathbf{y};\mathbf{u}_\infty,\lambda)](\mathbf{v}^{(0)}\cdot\nabla\mathbf{v}^{(0)})\, d\mathbf{y} = \mathbf{L}[\mathbf{x};-\mathbf{I}_0(\mathbf{z},\lambda)], \quad \mathbf{I}_0(\mathbf{z},\lambda) = \int_D \mathbf{H}(\mathbf{z},\mathbf{y};\mathbf{u}_\infty,\lambda)(\mathbf{v}^{(0)}\cdot\nabla\mathbf{v}^{(0)})\, d\mathbf{y} \qquad (4.1)$$

The integral $\mathbf{I}_0(\mathbf{z},\lambda)$ appearing in the above formula has the same asymptotics as $\mathbf{I}_1$, i.e.

$$I_{0i}(\mathbf{z},\lambda) = \int_D H_{ij}(\mathbf{z},\mathbf{y};u_\infty,0)\, v_k^{(0)} \frac{\partial v_j^{(0)}}{\partial y_k}\, d\mathbf{y} + B_i\lambda\ln\frac{1}{\lambda} + O(\lambda)$$

Substituting this expression into (4.1), we obtain

$$I_{2i}(\mathbf{x}, \lambda) = f_i^{(0)}(\mathbf{x}) + g_i^{(0)}(\mathbf{x})\,\lambda \ln \frac{1}{\lambda} + O(\lambda) \tag{4.2}$$

where $f_i^{(0)}(\mathbf{x})$ and $g_i^{(0)}(\mathbf{x})$ are solutions of the boundary value problems for the system of Oseen equations with boundary conditions

$$f_i^{(0)}(\mathbf{x})\,|_{\mathbf{x}\in S} = -\int_D H_{ij}(\mathbf{x}-\mathbf{y}, 0)\, v_k^{(S)}(\mathbf{y}) \frac{\partial v_j^{(S)}}{\partial y_k}(\mathbf{y})\, d\mathbf{y}, \quad \lim_{|\mathbf{x}|\to\infty} f_i^{(0)}(\mathbf{x}) = 0$$

$$g_i^{(0)}(\mathbf{x})\,|_{\mathbf{x}\in S} = -B_i, \quad \lim_{|\mathbf{x}|\to\infty} g_i^{(0)}(\mathbf{x}) = 0$$

Taking into account the first formula of (3.6) we can replace in (4.2), the solutions $f_i^{(0)}(\mathbf{x})$ and $g_i^{(0)}(\mathbf{x})$ of the Oseen boundary value problems by the corresponding solutions $f_i^{(S)}(\mathbf{x})$ and $g_i^{(S)}(\mathbf{x})$ of the boundary value problems for the Stokes equations. Combining the formulas (3.5) and (4.2) in this manner, we obtain

$$\mathbf{v}^{(1)}(\mathbf{x}, \lambda) = \mathbf{w}_0^{(S)}(\mathbf{x}) + \mathbf{w}_1^{(S)}(\mathbf{x})\,\lambda \ln \frac{1}{\lambda} + O(\lambda) \tag{4.3}$$

where $\mathbf{w}_0^{(S)}(\mathbf{x})$ denotes the solution of the inhomogeneous system of Stokes equations

$$\Delta \mathbf{v} - 2\lambda\, \mathrm{grad}\, p = v_k^{(S)} \frac{\partial \mathbf{v}^{(S)}}{\partial x_k}, \quad \mathrm{div}\, \mathbf{v} = 0 \tag{4.4}$$

with boundary conditions $\mathbf{v}(\mathbf{x})\,|_{\mathbf{x}\in S} = 0, \ \lim_{|\mathbf{x}|\to\infty} \mathbf{v}(\mathbf{x}) = 0$, and $\mathbf{w}_1^{(S)}(\mathbf{x})$ is a solution of the homogeneous system of Stokes equations with boundary conditions

$$\mathbf{w}_1^{(S)}(\mathbf{x})\,|_{\mathbf{x}\in S} = 0, \quad \lim_{|\mathbf{x}|\to\infty} \mathbf{w}_1^{(S)}(\mathbf{x}) = \mathbf{B} \tag{4.5}$$

Substituting (4.3) into the first formula of (1.3), we arrive at the following theorem.

Theorem 3. If the surface S of the body B has a curvature satisfying the Hölder condition and the function $\mathbf{u}_0(\mathbf{x})$ is twice continuously differentiable, then for $|\mathbf{x}| \ll \lambda^{-1}$ and sufficiently small λ the solution of the boundary problem (1.1) is represented by the asymptotic formula

$$u_i(\mathbf{x}, \lambda) = u_i^{(0)}(\mathbf{x}, \lambda) + 2\lambda w_{0i}^{(S)}(\mathbf{x}) + 2\lambda^2 \ln \frac{1}{\lambda}\, w_{1i}^{(S)}(\mathbf{x}) + O(\lambda^2) \tag{4.6}$$

$$(i = 1, 2, 3)$$

where $u_i^{(0)}(\mathbf{x}, \lambda)$ is the solution of the boundary value problem for the system of Oseen equations with boundary conditions from (1.1).

5. A flow of viscous fluid past a sphere was solved by Proudman and Pearson in [6] using the method of matched asymptotic expansions. Applying the formula (4.6) to this problem, we obtain an internal expansion of the Proudman and Pearson solution. Thus the above internal expansion acquires a vigorous proof within the specified range of values of the Reynolds number.

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